

NEW RESULTS ON THE REAL JACOBIAN CONJECTURE

JAUME LLIBRE

The real Jacobian conjecture said: *If $F = (f, g) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a polynomial map such that $\det(DF(x))$ is different from zero for all $x \in \mathbb{R}^2$, then F is injective.*

This conjecture had a negative answer by Pinchuk in 1994. Now several authors look for adding an additional assumption to the fact that $\det(DF(x))$ is different from zero for all $x \in \mathbb{R}^2$, in order that the conjecture holds.

The next two theorema are proved using qualitative theory of the ordinary differential equations in the plane. More precisely in the talk we will show how the Poincaré compactification of polynomial vector fields, and the Poincaré–Hopf Theorem are used for proving the next two results.

Theorem 1. *Let $F = (f, g) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a polynomial map such that $\det DF(x)$ is different from zero for all $x \in \mathbb{R}^2$. We assume that the degrees of f and g are equal and that the higher homogeneous terms of the polynomials f and g do not have real linear factors in common, then F is injective.*

Theorem 2. *Let $F = (f, g) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a polynomial map such that $\det DF(x)$ is different from zero for all $x \in \mathbb{R}^2$ and $F(0, 0) = (0, 0)$. If the higher homogeneous terms of the polynomials $ff_x + gg_x$ and $ff_y + gg_y$ do not have real linear factors in common, then F is injective.*

DEPARTAMENT DE MATEMÀTIQUES UNIVERSITAT AUTÒNOMA DE BARCELONA,
08193 BELLATERRA, BARCELONA, CATALONIA, SPAIN

E-mail address: `jllibre@mat.uab.cat`